COOLING OF CASTINGS IN SAND MOLDS

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The problem of the cooling of flat, cylindrical, and spherical castings in sand molds is solved. The specific heat of the liquid metal, the heat of crystallization, and the heat of pearlite transformation were considered in relation to the increase in the initial temperature. Since the solution for a cylinder has already been tabulated, this paper gives a table of numerical values for a sphere only.

In the development of casting technology it is particularly important to know the kinetics of cooling of castings. In this connection, analytical solutions of the problem of the duration of cooling of a casting of an infinite plate in a sand mold have recently appeared [1–3]. These solutions were derived on the following assumptions:

1. The thermophysical constants of the metal and mold are constant and are independent of the temperature.
2. The heat of solidification, the heat of pearlite transformation, and the heat content of the liquid metal can be replaced by the equivalent heat content of the solid metal.
3. The temperature gradient in the metal is zero.

The second assumption means that the cooling of a solidifying casting can be regarded as the cooling of a body with initial temperature

$$t_s = t_c + [c'_{\text{м}}(t_p - t_c) + q_c + q_p]/c_1 .$$

The error introduced by these assumptions has been determined from [1–3] for a plate on a hydrostatic integrator and does not exceed 5% for the period of solidification of the casting.

In this paper the corresponding problems for cylindrical and spherical castings are considered with the above assumptions. The determination of the temperature distribution in a sand mold necessitates the solution of the same differential equation with the same boundary conditions as in [4], the only difference being that the problem there was solved for $0 \le \xi \le 1$, whereas in this problem the case $1 \le \xi \le \infty$ must be considered. Thus, the temperature distribution in the mold is given by the equation

$$\frac{\partial \Theta(\xi, \text{Fo})}{\partial \text{Fo}} = \frac{\partial^2 \Theta(\xi, \text{Fo})}{\partial \xi^2} + \frac{m}{\xi}\,\frac{\partial \Theta(\xi, \text{Fo})}{\partial \xi},$$

$$\text{Fo} \geqslant 0, \quad 1 \leqslant \xi \leqslant \infty \tag{1}$$

with the boundary conditions

$$\Theta(\xi, 0) = 0, \quad \Theta(1, 0) = 1,$$

$$-K\,\frac{\partial \Theta(1, \text{Fo})}{\partial \xi} + \frac{\partial \Theta(1, \text{Fo})}{\partial \text{Fo}} = 0, \tag{2}$$

where

$$\Theta = [t(x, \tau) - t_0]/(t_s - t_0); \quad \xi = r/R;$$

$$\text{Fo} = a\tau/R^2; \quad K = c\gamma FR/c_{\text{м}}\gamma_{\text{м}} V_{\text{м}},$$

m = 0, 1, and 2, respectively, for a plate, cylinder, and sphere.

We consider the cooling of a flat wall of thickness 2R. The solution for the temperature image is obtained in the form

$$\bar{\Theta}(\xi, s) = \exp[-(\xi - 1)\sqrt{s}]/\sqrt{s}(K + \sqrt{s}). \tag{3}$$

The original is found from the table of images [5]:

$$\Theta(\xi, \text{Fo}) = \exp[(\xi - 1)K + (K\sqrt{\text{Fo}})^2]\,\text{erfc}\,[(\xi - 1)/2\sqrt{\text{Fo}} + K\sqrt{\text{Fo}}]. \tag{4}$$

When $\xi = 1$ the temperature of the metal is obtained, since at any instant τ it is equal to the surface temperature of the mold,

$$\Theta_{\text{м}}(\text{Fo}) = \exp(K\sqrt{\text{Fo}})^2\,\text{erfc}\,(K\sqrt{\text{Fo}}). \tag{5}$$

This solution is given in [1, 3].

We consider a cylinder of radius R. The solution for the temperature image is obtained in the form

$$\bar{\Theta}(\xi, s) = K_0(\xi\sqrt{s})/[sK_0(\sqrt{s}) + K\sqrt{s}K_1(\sqrt{s})]. \tag{6}$$

Using the inverse Laplace transform and converting from an integral of a complex variable to an integral of a real variable, we obtain

$$\Theta(\xi, \text{Fo}) = \frac{2}{\pi} \times$$

$$\times \int_0^\infty \frac{J_0(\mu\xi)[\mu Y_0(\mu) - KY_1(\mu)] - Y_0(\mu\xi)[\mu J_0(\mu) - KJ_1(\mu)]}{[\mu J_0(\mu) - KJ_1(\mu)]^2 + [\mu Y_0(\mu) - KY_1(\mu)]^2} \times$$

$$\times \exp(-\mu^2 \text{Fo})\, d\mu. \tag{7}$$

When $\xi = 1$ we obtain the temperature of the metal

$$\Theta_{\text{м}}(\text{Fo}) = \frac{4K}{\pi^2} \times$$

$$\times \int_0^\infty \frac{\exp(-\mu^2 \text{Fo})}{[\mu J_0(\mu) - KJ_1(\mu)]^2 - [\mu Y_0(\mu) - KY_1(\mu)]^2}\,\frac{d\mu}{\mu}. \tag{8}$$

Formula (8) has been used to measure the thermal conductivity of rocks [6]. The most interesting of the investigations known to the author is [7], where the integral of formula (8) is tabulated.

Values of Metal Temperature when $\xi = 1$

Fo	Θ_M for K equal to						
	0.5	1	1.5	2	4	6	8
0.2	0.7210	0.5437	0.4257	0.3438	0.1822	0.1193	0.0876
0.3	0.6577	0.4638	0.3460	0.2699	0.1336	0.0856	0.0623
0.4	0.6059	0.4041	0.2902	0.2206	0.1046	0.0663	0.0481
0.5	0.5620	0.3569	0.2485	0.1853	0.0852	0.0537	0.0389
0.6	0.5237	0.3185	0.2161	0.1586	0.0715	0.0449	0.0325
0.7	0.4898	0.2865	0.1902	0.1379	0.0612	0.0383	0.0278
0.8	0.4596	0.2595	0.1690	0.1213	0.0532	0.0333	0.0242
0.9	0.4324	0.2363	0.1514	0.1077	0.0469	0.0294	0.0213
1	0.4076	0.2162	0.1365	0.0965	0.0418	0.0262	0.0190
2	0.2456	0.1061	0.0622	0.0428	0.0184	0.0116	0.0085
3	0.1624	0.0631	0.0363	0.0250	0.0110	0.0070	0.0051
4	0.1139	0.0419	0.0242	0.0168	0.0075	0.0048	0.0035
5	0.0834	0.0300	0.0174	0.0122	0.0055	0.0036	0.0026
6	0.0633	0.0227	0.0133	0.0094	0.0043	0.0028	0.0020
7	0.0495	0.0178	0.0106	0.0075	0.0034	0.0022	0.0017
8	0.0396	0.0144	0.0086	0.0061	0.0028	0.0019	0.0014
9	0.0324	0.0120	0.0072	0.0052	0.0024	0.0016	0.0012
10	0.0270	0.0101	0.0062	0.0044	0.0021	0.0014	0.0010
15	0.0132	0.0053	0.0033	0.0024	0.0012	0.0008	0.0006
20	0.0080	0.0034	0.0021	0.0016	0.0008	0.0005	0.0004

We consider a sphere of radius R. The solution for the temperature image is obtained in the form

$$\xi\bar{\Theta}(\xi, s) = \exp[-(\xi-1)\sqrt{s}]/[s + K(1+\sqrt{s})]. \quad (9)$$

Using the inverse Laplace transform and converting from an integral of a complex variable to an integral of a real variable, we obtain

$$\xi\Theta(\xi, \mathrm{Fo}) = \frac{2}{\pi}\int_0^\infty \frac{(K-\mu^2)\sin\mu(\xi-1) + K\mu\cos\mu(\xi-1)}{(K-\mu^2)^2 + (K\mu)^2} \times \mu\exp(-\mu^2\mathrm{Fo})\,d\mu. \quad (10)$$

When $\xi = 1$ the temperature of the metal is obtained,

$$\Theta_M(\mathrm{Fo}) = \frac{2}{\pi K}\int_0^\infty \frac{\exp(-\mu^2\mathrm{Fo})}{1 + (1/\mu - \mu/K)^2}\,d\mu. \quad (11)$$

The results of tabulation of this integral on a Minsk-2 electronic digital computer are given in Table 1 for the same values of the parameters as were chosen by Jaeger for a cylinder [7].

NOTATION

t_c is the temperature of crystallization; t_p is the temperature of pouring; c'_M is the specific heat of liquid metal; q_c is the specific heat of crystallization; q_p is the specific heat of pearlite transformation; c_1 is the specific heat of solid metal.

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